

# An Improved Supersymmetric SU(5)<sup>1</sup>

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## Abstract

By supplementing minimal supersymmetric  $SU(5)$  (MSSU(5)) with a flavor  $\mathcal{U}(1)$  symmetry and two pairs of  $\overline{15} + 15$  ‘matter’ supermultiplets, we present an improved model which explains the charged fermion mass hierarchies and the magnitudes of the CKM matrix elements, while avoiding the undesirable asymptotic mass relations  $m_s = m_\mu$ ,  $\frac{m_d}{m_s} = \frac{m_e}{m_\mu}$ . The strong coupling  $\alpha_s(M_Z)$  is predicted to be approximately 0.115, and the proton lifetime is estimated to be about five times larger than the MSSU(5) value. The atmospheric and solar neutrino puzzles are respectively resolved via maximal  $\nu_\mu - \nu_\tau$  and small mixing angle  $\nu_e - \nu_s$  MSW oscillations, where  $\nu_s$  denotes a sterile neutrino. The  $\mathcal{U}(1)$  symmetry ensures not only a light  $\nu_s$  but also automatic ‘matter’ parity.

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# 1 Introduction

It is well known that MSSU(5) [1] provides no explanation of the charged fermion mass hierarchies and mixings, predicts the undesirable asymptotic relations  $m_s = m_\mu$ ,  $\frac{m_d}{m_s} = \frac{m_e}{m_\mu}$ , and cannot simultaneously account for the atmospheric and solar neutrino data. In addition, taking account of supersymmetric and heavy threshold corrections, the MSSU(5) value for the strong coupling is 0.126 [2], to be compared with the world average value of  $0.117 \pm 0.005$  [3]

In a recent paper [4] we considered an  $SU(5)$  model supplemented by two key ingredients. One is a  $\mathcal{U}(1)$  flavor symmetry [5], suitably implemented for explaining the charged fermion mass hierarchies and mixings, and consistent with a variety of neutrino oscillation scenarios. For instance, it was shown how bi-maximal neutrino mixings could be realized for explaining the atmospheric and solar neutrino data. A second key ingredient is the introduction of two pairs of vector-like ‘matter’ superfields belonging to the  $\overline{15} + 15$  representations of  $SU(5)$ . They play an essential role in avoiding the undesirable asymptotic mass relations mentioned above <sup>4</sup>.

The purpose of this paper is to explore some key phenomenological consequences of such an extended  $SU(5)$  scheme. In particular, it turns out that the  $\overline{15} + 15$  superfields play an essential role in reducing the predicted strong coupling to  $\simeq 0.115$ , which is in excellent agreement with experiments. Furthermore, they also have an impact, albeit a modest one, on the proton lifetime. It turns out to be about five times longer than the MSSU(5) value.

For obtaining a natural understanding of the charged fermion mass hierarchies and magnitudes of the CKM matrix elements, we supplement the  $\mathcal{U}(1)$  flavor symmetry with a  $Z_2$   $\mathcal{R}$ -symmetry. The latter helps in the generation of desired mass scales. The resolution of the atmospheric and solar neutrino puzzles necessitates in this approach the introduction of a sterile neutrino state  $\nu_s$  which, thanks to the  $\mathcal{U}(1)$  symmetry, can be kept light. Maximal  $\nu_\mu - \nu_\tau$  oscillations resolve the atmospheric neutrino anomaly, while the small mixing angle  $\nu_e - \nu_s$  MSW oscillations can explain the solar neutrino data. It turns out that the  $\mathcal{U}(1)$  symmetry also implies an automatic  $Z_2$  ‘matter’ parity (including higher order terms).

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<sup>4</sup>The extended ‘matter’ sector of  $SU(5)$ , with additional  $\overline{15} + 15$  supermultiplets, leads to a scenario quite different from the case which includes scalar 45-plets [6].

## 2 Extended Supersymmetric $SU(5)$ : Charged Fermion Masses and Mixings

The scalar sector of  $SU(5)$ , which we consider here, in addition to  $\Sigma(24)$ ,  $\bar{H}(\bar{5})$ ,  $H(5)$  multiplets, also contains  $S$  and  $X$  singlets. We introduce the symmetry  $Z_2 \times \mathcal{U}(1)$ , where  $Z_2$  is an  $\mathcal{R}$ -symmetry. Under  $Z_2$ ,

$$\begin{aligned} (\Sigma, \bar{H}, H, S) &\rightarrow -(\Sigma, \bar{H}, H, S) , \\ X &\rightarrow X , \quad W \rightarrow -W . \end{aligned} \tag{1}$$

As will be discussed in more detail below, the anomalous  $\mathcal{U}(1)$  flavor symmetry is crucial for obtaining the hierarchies among fermion masses and mixings. The  $\mathcal{U}(1)$  charges of the ‘scalars’ are:

$$\begin{aligned} Q_X &= 1 , \quad Q_{\bar{H}} = -Q_H = 2r , \\ Q_\Sigma &= Q_S = 0 , \end{aligned} \tag{2}$$

( $r$  is undetermined for the time being).

The most general renormalizable scalar superpotential allowed under the symmetries reads:

$$\begin{aligned} W_S &= -\Lambda^2 S + \frac{\lambda}{3} S^3 + \frac{h}{2} S \text{Tr} \Sigma^2 + \frac{\sigma}{3} \text{Tr} \Sigma^3 + \\ &\quad \bar{H}(\lambda_1 S + \lambda_2 \Sigma) H , \end{aligned} \tag{3}$$

where  $\lambda$ ,  $h$ ,  $\sigma$  and  $\lambda_{1,2}$  are dimensionless couplings, and  $\Lambda$  is a mass scale of order  $M_{GUT} \equiv M_G$ . From (3), with supersymmetry unbroken, one obtains a non-vanishing  $\langle \Sigma \rangle$  (and also  $\langle S \rangle$ ) in the desirable direction

$$\langle \Sigma \rangle = \text{Diag}(2, 2, 2, -3, -3) \cdot V , \tag{4}$$

with

$$V = \frac{h\Lambda}{(15h^3 + \lambda\sigma^2)^{1/2}} , \quad \langle S \rangle = \frac{\sigma\Lambda}{(15h^3 + \lambda\sigma^2)^{1/2}} . \tag{5}$$

From (5), assuming that  $\Lambda \sim 10^{16}$  GeV, with all coupling constants of order unity, we have

$$\frac{V}{M_P} \sim \frac{\langle S \rangle}{M_P} \equiv \epsilon_G \simeq 10^{-2} . \tag{6}$$

As for the flavor  $\mathcal{U}(1)$  symmetry, it is natural to consider it as an anomalous gauge symmetry. It is well known that anomalous  $U(1)$  factors can appear in effective field theories from strings. The cancellation of its anomalies occurs through the Green-Schwarz mechanism [7]. Due to the anomaly, the Fayet-Iliopoulos term

$$\xi \int d^4\theta V_A \quad (7)$$

is always generated [8], where, in string theory,  $\xi$  is given by [9]

$$\xi = \frac{g_A^2 M_P^2}{192\pi^2} \text{Tr} Q . \quad (8)$$

The  $D_A$ -term will have the form

$$\frac{g_A^2}{8} D_A^2 = \frac{g_A^2}{8} \left( \sum Q_a |\varphi_a|^2 + \xi \right)^2 , \quad (9)$$

where  $Q_a$  is the ‘anomalous’ charge of  $\varphi_a$  superfield.

In ref. [10] the anomalous  $\mathcal{U}(1)$  symmetry was considered as a mediator of SUSY breaking, while in ref. [11], the anomalous Abelian symmetries were exploited as flavor symmetries for a natural understanding of hierarchies of fermion masses and mixings.

In our  $SU(5)$  model, assuming  $\text{Tr} Q < 0$  ( $\xi < 0$ ) and taking into account (2), we can ensure that the cancellation of (9) fixes the VEV of  $X$  field as:

$$\langle X \rangle = \sqrt{-\xi} . \quad (10)$$

Further, we will assume that

$$\frac{\langle X \rangle}{M_P} \equiv \epsilon \simeq 0.2 . \quad (11)$$

The parameter  $\epsilon$  is an important expansion parameter for understanding the magnitudes of fermion masses and mixings.

Together with the  $(10 + \bar{5})_i$  ( $i = 1, 2, 3$  is a family index) matter multiplets, we consider two pairs  $(\bar{15} + 15)_{1,2}$  of ‘matter’, which will play an important role for obtaining acceptable pattern of fermion masses. The transformation properties of ‘matter’ superfields under  $\mathcal{U}(1)$  are given in Table (1). The relevant couplings will be<sup>5</sup>:

$$\begin{array}{c} 10_1 \\ 10_2 \\ 10_3 \end{array} \begin{pmatrix} \bar{5}_1 & \bar{5}_2 & \bar{5}_3 \\ \epsilon^5 & \epsilon^4 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \overline{H} \epsilon^a , \quad (12)$$

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<sup>5</sup>We assume that  $Z_2$   $\mathcal{R}$  symmetry does not act on the matter superfields.

Table 1: Transformation properties of matter superfields under  $\mathcal{U}(1)$  symmetry

	$10_3$	$10_2$	$10_1$	$\bar{5}_i$	$15_2$	$\bar{15}_2$	$15_1$	$\bar{15}_1$
$\mathcal{U}(1)$	$r$	$r-1$	$r-3$	$-(3r+a+3-i)$	$r-2$	$1-r$	$r-3$	$3-r$

$$\begin{matrix} 10_1 & 10_2 & 10_3 \\ 10_1 \left( \begin{array}{ccc} \epsilon^6 & \epsilon^4 & \epsilon^3 \\ \epsilon^4 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon & 1 \end{array} \right) & & \end{matrix} H , \quad (13)$$

$$\begin{matrix} \bar{5}_1 & \bar{5}_2 & \bar{5}_3 \\ 15_1 \left( \begin{array}{ccc} \epsilon^5 & \epsilon^4 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon^2 \end{array} \right) & & \end{matrix} \bar{H} \epsilon^a , \quad (14)$$

$$\begin{matrix} \bar{15}_1 & \bar{15}_2 \\ \bar{15}_1 \left( \begin{array}{ccc} 1 & 0 & 0 \\ \epsilon^2 & 1 & 0 \end{array} \right) \Sigma , & \bar{15}_2 \left( \begin{array}{cc} 15_1 & 15_2 \\ 1 & 0 \\ \epsilon^2 & \epsilon \end{array} \right) (\Sigma + S). \end{matrix} \quad (15)$$

Noting that in terms of  $SU(3)_c \times SU(2)_W$ ,  $15 = (3, 2) + (6, 1) + (1, 3)$ , we may conclude that the couplings involving  $15, \bar{15}$  do not affect  $e^c$  and  $l$  states from  $10$  and  $\bar{5}$  respectively (they can only affect the  $q$  states). The lepton mass matrix will coincide with (12), from which we have:

$$\lambda_\tau \sim \epsilon^a , \quad \lambda_e : \lambda_\mu : \lambda_\tau \sim \epsilon^5 : \epsilon^2 : 1 , \quad (16)$$

where  $a = 0, 1, 2$  determines the value of  $\tan \beta (\sim \frac{m_t}{m_b} \epsilon^a)$ .

Turning to the quark sector, from (15) we see that  $10_3$ -plet also is not affected, while  $q_{10_1}, q_{10_2}$  will be mixed with  $q_{15_1}, q_{15_2}$ . Analyzing (15), one can easily verify that for the ‘light’  $q_i$  states we will have:

$$\begin{aligned} (10_1, 15_1) &\supseteq q_1 , & 15_2 &\supseteq q_2 , \\ 10_2 &\supseteq \epsilon q_2 , & 10_3 &\supseteq q_3 . \end{aligned} \quad (17)$$

From (17), (12) and (14), we find the down quark mass matrix to be

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \begin{pmatrix} d_1^c & d_2^c & d_3^c \\ \epsilon^5 & \epsilon^4 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \epsilon^a h_d , \quad (18)$$

from which

$$\lambda_b \sim \epsilon^a , \quad \lambda_d : \lambda_s : \lambda_b \sim \epsilon^5 : \epsilon^3 : 1 . \quad (19)$$

From (12), (16), (18), (19), and taking into account (17), we obtain

$$\lambda_b = \lambda_\tau \left( 1 + \mathcal{O}(\epsilon^2) \right) \sim \epsilon^a , \quad (20)$$

while, for Yukawas of the second generation,

$$\lambda_s \sim \epsilon \lambda_\mu \simeq \frac{1}{5} \lambda_\mu . \quad (21)$$

Assuming that  $\lambda_d \sim 2\lambda_e$ , from (16) (19) and (21) we will have

$$\frac{\lambda_s}{\lambda_d} \sim \frac{1}{10} \frac{\lambda_\mu}{\lambda_e} \simeq 20 . \quad (22)$$

For up-type quarks, from (13), taking into account (17), we obtain

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \begin{pmatrix} u_1^c & u_2^c & u_3^c \\ \epsilon^6 & \epsilon^4 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon & 1 \end{pmatrix} h_u , \quad (23)$$

from which we obtained the desired Yukawa couplings

$$\lambda_t \sim 1 , \quad \lambda_u : \lambda_c : \lambda_t \sim \epsilon^6 : \epsilon^3 : 1 . \quad (24)$$

From (18) and (23), for the CKM matrix elements we find

$$V_{us} \sim \epsilon , \quad V_{cb} \sim \epsilon^2 , \quad V_{ub} \sim \epsilon^3 , \quad (25)$$

in good agreement with observations.

To conclude, we see that with the help of  $\mathcal{U}(1)$  flavor symmetry and  $\overline{15} + 15$ -plets, in addition to the desirable hierarchies of charged fermion masses and CKM mixing angles, we can also get reasonable [(20), (21), (22)] asymptotic relations.

### 3 Value of $\alpha_s(M_Z)$

By analyzing the spectra of decoupled heavy states, from (15) we can verify that the masses of the states  $(\bar{6}, 1) + (1, \bar{3}) + (6, 1) + (1, 3)$  (from  $\bar{15}_2 + 15_2$  respectively) are below the GUT scale and equal to  $M_S \simeq M_G \epsilon$ . Indeed, these states will change the running of the gauge couplings above the  $M_S$  scale and, as we will see, this opens up the possibility to obtain a reduced value for  $\alpha_s(M_Z)$  <sup>6</sup>.

The solutions of the three renormalization-group (RG) equations are [2]

$$\alpha_G^{-1} = \alpha_a^{-1} - \frac{b_a}{2\pi} \ln \frac{M_G}{M_Z} - \frac{b'_a}{2\pi} \ln \frac{M_G}{M_S} + \Delta_a + \delta_a, \quad (26)$$

where  $\alpha_G$  is the gauge coupling at the GUT scale,  $\alpha_a$  denote the gauge couplings at  $M_Z$  scale ( $\alpha_{1,2,3}$  are the gauge couplings of  $U(1)_Y$ ,  $SU(2)_W$  and  $SU(3)_c$  respectively), while  $b_a, b'_a$  are given by

$$(b_1, b_2, b_3) = \left(\frac{33}{5}, 1, -3\right), \quad (b'_1, b'_2, b'_3) = \left(\frac{34}{5}, 4, 5\right). \quad (27)$$

The  $\Delta_a$  include all possible SUSY and heavy threshold corrections, and contributions from the two loop effects of MSSU(5).  $\delta_a$  denote the difference of gauge coupling running between MSSU(5) and present model from  $M_S$  up to  $M_G$  in two loop approximation,

$$\delta_a = \frac{1}{4\pi} \left( \frac{b_{ab} + b'_{ab}}{b_b + b'_b} \ln \frac{\alpha_b(M_S)}{\alpha_G} - \frac{b_{ab}}{b_b} \ln \frac{\alpha_b(M_S)}{\alpha_G^0} \right), \quad (28)$$

where

$$b_{ab} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix}, \quad b'_{ab} = \begin{pmatrix} \frac{904}{75} & \frac{144}{5} & \frac{128}{3} \\ \frac{144}{15} & 24 & 0 \\ \frac{16}{3} & 0 & \frac{128}{3} \end{pmatrix} \quad (29)$$

and the appropriate couplings in (28) are calculated in one loop approximation.  $\alpha_G^0$  is the gauge coupling at  $M_G$  in MSSU(5).

From (26), taking into account (27), one finds

$$\alpha_s^{-1} = (\alpha_s^{-1})^0 + \frac{3}{2\pi} \ln \frac{M_G}{M_S} + \delta, \quad (30)$$

where  $(\alpha_s^{-1})^0 = \frac{1}{7} (12\alpha_w^{-1} - 5\alpha_Y^{-1}) + \frac{1}{7} (12\Delta_2 - 5\Delta_1 - 7\Delta_3)$  corresponds to the value of  $\alpha_s$  obtained in MSSU(5) case, and  $\delta = \frac{1}{7} (12\delta_2 - 5\delta_1 - 7\delta_3)$ . Using the result  $(\alpha_s^{-1})^0 = 1/0.126$

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<sup>6</sup>For alternative mechanisms of achieving this see [12].

[2], and taking  $M_S/M_G \simeq \epsilon \simeq 0.2$ , (neglecting  $\delta$  for the time being), we obtain  $\alpha_s \simeq 0.115$ , in good agreement with experimental data [3]. Taking into account (26) and (29), from (28) we obtain  $\delta = -0.015$ , thus leaving the value of  $\alpha_s$  unchanged as expected.

## 4 Proton Decay

From (12) and (14), taking into account (17), we see that  $ql\bar{T}$  type couplings in the family space have the same hierarchical structure as the down quark mass matrix (18). As far as  $qqT$  operators are concerned, from (13), (17) one obtains,

$$\begin{matrix} & q_1 & q_2 & q_3 \\ \begin{matrix} q_1 \\ q_2 \\ q_3 \end{matrix} & \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} & T, \end{matrix} \quad (31)$$

from which we see that the appropriate couplings are suppressed by a factor  $\epsilon (\sim 1/5)$  compared to the up type quark mass matrix (23). From (26), we find that  $M_G = \left(\frac{M_S}{M_G}\right)^{1/2} M_G^0 \simeq M_G^0/\sqrt{5}$ , where  $M_G^0$  is the GUT scale in MSSU(5). From all this we may conclude that the proton life time in our model will be  $\tau_p \sim 5 \cdot \tau_p^0$  (that is, a factor 5 larger than in MSSU(5)). For further suppression of nucleon decay, the mass scale  $M_S$  should be reduced. However, this would ruin the gauge coupling unification unless some additional mechanism (for retaining unification) is applied. Such a program can be successfully realized in extended  $SU(5+N)$  GUTs [13].

## 5 Neutrino Oscillations

Turning to the neutrino sector, for accommodating the recent solar and atmospheric Superkamiokande data (see [14], [15] respectively), we will invoke the mechanism suggested in refs. [16, 4]. The atmospheric anomaly is explained through maximal  $\nu_\mu - \nu_\tau$  mixings which is achieved through quasi-degenerate massive  $\nu_\mu, \nu_\tau$  states. Since these states are too heavy to explain the solar neutrino data, we are led introduce a sterile neutrino state  $\nu_s$ . The solar neutrino anomaly is resolved via the small angle  $\nu_e - \nu_s$  MSW oscillations.

Together with  $\nu_s$  state we introduce two heavy right handed states  $\mathcal{N}_{2,3}$ . Choosing the  $\mathcal{U}(1)$  charges of these states to be

$$Q_{\mathcal{N}_2} = -\frac{1}{2}, \quad Q_{\mathcal{N}_3} = \frac{1}{2}, \quad Q_{\nu_s} = -\frac{41}{2}, \quad (32)$$

and in Table (1) taking



$$r = -\frac{a}{5} - \frac{1}{10} , \quad (33)$$

the relevant couplings are (these singlet states do not transform under the  $Z_2$   $\mathcal{R}$  symmetry):

$$\begin{pmatrix} \bar{5}_1 \\ \bar{5}_2 \\ \bar{5}_3 \end{pmatrix} \begin{pmatrix} \mathcal{N}_2 & \mathcal{N}_3 \\ \epsilon^2 & \epsilon \\ \epsilon & 1 \\ 1 & 0 \end{pmatrix} H , \quad \begin{pmatrix} \mathcal{N}_2 \\ \mathcal{N}_3 \end{pmatrix} \begin{pmatrix} \epsilon & 1 \\ 1 & 0 \end{pmatrix} \rho S , \quad (34)$$

$$W_{\nu_s} = \epsilon^{20} (\bar{5}_3 + \epsilon \bar{5}_2 + \epsilon^2 \bar{5}_1) \nu_s H + S \epsilon^{41} \nu_s^2 , \quad (35)$$

where  $\rho$  is a dimensionless coupling. Integration of  $\mathcal{N}_{2,3}$  states leads to the mass matrix for the ‘light’ neutrinos:

$$m_\nu = \begin{pmatrix} \nu_s \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \begin{pmatrix} \nu_s & \nu_e & \nu_\mu & \nu_\tau \\ m_{\nu_s} & m' \epsilon^2 & m' \epsilon & m' \\ m' \epsilon^2 & m \epsilon^3 & m \epsilon^2 & m \\ m' \epsilon & m \epsilon^2 & m \epsilon & m \\ m' & m \epsilon & m & 0 \end{pmatrix} , \quad (36)$$

where we have defined:

$$m \equiv \frac{h_u^2}{\rho M_P \epsilon_G} , \quad m' \equiv \epsilon^{20} h_u , \quad m_{\nu_s} \equiv M_P \epsilon_G \epsilon^{41} . \quad (37)$$

Taking  $\rho \sim 2 \cdot 10^{-2}$ ,  $\epsilon = 0.2 - 0.22$ , from (37) we have

$$\begin{aligned} m &\simeq 6.3 \cdot 10^{-2} \text{eV} , \\ m_{\nu_s} &= (5 \cdot 10^{-4} - 3 \cdot 10^{-2}) \text{eV} , \\ m' &= (1.8 \cdot 10^{-3} - 1.2 \cdot 10^{-2}) \text{eV} . \end{aligned} \quad (38)$$

Note that the sterile neutrino is kept light (see (35), (38)) by the  $\mathcal{U}(1)$  symmetry [17, 16]. Taking

$$m = 6.3 \cdot 10^{-2} \text{eV} , \quad m' = 1.8 \cdot 10^{-3} \text{eV} , \quad m_{\nu_s} = 2 \cdot 10^{-3} \text{eV} \quad (39)$$

from (39) and (36), we have for the atmospheric neutrino oscillation parameters

$$\Delta m_{23}^2 = 2m^2 \epsilon \simeq 2 \cdot 10^{-3} \text{ eV}^2 ,$$

$$\sin^2 2\theta_{\mu\tau} = 1 - \mathcal{O}(\epsilon^2) . \quad (40)$$

The solar neutrino oscillation parameters are given by

$$\begin{aligned} \Delta m_{\nu_e \nu_s}^2 &\simeq m_{\nu_s}^2 \sim 4 \cdot 10^{-6} \text{ eV}^2 , \\ \sin^2 2\theta_{es} &\simeq 4 \left( \frac{m' \epsilon^2}{m_{\nu_s}} \right)^2 \sim 5 \cdot 10^{-3} . \end{aligned} \quad (41)$$

We see that the  $\mathcal{U}(1)$  flavor symmetry helps provide a natural explanation of the solar and atmospheric experimental data. Note that  $a$  is still undetermined, and therefore the magnitude of  $\tan \beta$  is not fixed in our model.

## 6 Automatic Matter Parity

Let us conclude by considering all possible ‘matter’ parity violating operators:

$$\begin{aligned} &\bar{5}_i H , \quad 10_i \bar{H} (\Sigma \bar{H}) , \quad (\Sigma + S) 15_i \bar{H} \bar{H} , \\ &(\Sigma + S) \bar{15}_i H H , \quad (\Sigma + S) 10_i \bar{5}_j \bar{5}_k , \quad 10_i 10_j 10_k \bar{H} , \\ &(\Sigma + S) 15_i \bar{5}_j \bar{5}_k , \quad \Sigma^2 15_i 15_j 15_k \bar{H} , \quad \dots \end{aligned} \quad (42)$$

From Table (1), taking into account (33), we observe that the terms in (42) all have non-integer  $\mathcal{U}(1)$  charges, and consequently are forbidden to ‘all orders’ in powers of  $X$ . Therefore, thanks to  $\mathcal{U}(1)$  flavor symmetry, the model has automatic matter parity.

## 7 Conclusion

In conclusion, we note that the mechanisms discussed here for resolving the various puzzles in  $SU(5)$  can be successfully generalized to  $SU(5 + N)$  GUTs [13]. In this paper we have not addressed the gauge hierarchy problem whose resolution in  $SU(5)$  requires additional ‘scalar’ multiplets belonging to  $50 + \bar{50} + 75$ . In such a scenario the Higgs doublets remain ‘massless’, while the color triplets obtain masses by mixing with the triplets in  $50, \bar{50}$ . On the other hand, in order to retain perturbative gauge couplings up to  $M_P$ , the masses of  $50, \bar{50}$  states should exceed  $M_G$ , which means that the ordinary color triplets (from  $H, \bar{H}$ ) will lie below  $M_G$ . This would further destabilize the proton, and possibly disrupt unification of the gauge couplings. To avoid this, one could either consider more complicated  $SU(5)$  scenarios [18] with extended scalar sector, or extended  $SU(5 + N)$  GUTs [13]. In the latter case, for instance, a  $SU(6)$  model has been discussed in which the MSSM Higgs doublets are pseudo-Goldstone bosons, the proton lifetime is  $\sim 10^2 \tau_p^{SU(5)}$ , and neutrino oscillations involve bi-maximal mixings.

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